14MDE/MMD/MAR/MAU/IAE/MST/MTH/MTP/MTE/MTR/MEA/MCM/CAE11

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First Semester M.Tech. Degree Examination, Dec.2014/Jan.2015

Applied Mathematics

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Define : (i) Round off errors (ii) Significant figures (iii) Truncation errors
Round off the numbers 865250 and 37.46235 to four significant figures and find the relative error in each case. **(10 Marks)**

b. Derive the expression $V(t) = \frac{mg}{C}\left(1 - e^{-\left(\frac{C}{m}\right)t}\right)$ for a parachutist jumps out of a stationary hot air ballon. Compute the velocity prior to opening the chute when the mass is 68.1 kg, the drag coefficient is 12.5 kg/s, gravitational force is 9.8. (Take t = 4 sec as step size) **(10 Marks)**

2 a. Explain False position method to establish the roots of the equation f(x) = 0. Find two approximations when the root lies in (1.4, 1.5) for the equation $x^6 - x^4 - x^3 - 1 = 0$. **(10 Marks)**

b. Discuss Newton-Raphson method to find the root of the equation f(x) = 0. Find the root of the equation $x\log_{10} x - 1.2 = 0$. Take the initial of x as 2. Compute three approximations. **(10 Marks)**

3 a. Apply Bairstow's method to extract the quadratic factor $x^2 + px + q$ when $f(x) = x^3 + x^2 - x + 2 = 0$ with initial values of p and q as − 0.9 and 0.9 respectively. **(10 Marks)**

b. Use Graeffe's root squaring method (thrice) to find the roots of the equation, $f(x) = x^3 - 2x^2 - 5x + 6 = 0$ **(10 Marks)**

4 a. Give the equations for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ using Newton's forward and backward interpolation formulas. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at x = 1.5 given that, **(10 Marks)**

x	1.5	2.0	2.5	3.0	3.5	4.0
y	3.375	7.0	13.625	24.0	38.875	59.0

b. Use Romberg's method to compute $\int_0^1 \frac{dx}{1+x}$ correct to three decimal places. Use h = 0.5, 0.25, 0.125. Compare with direct integration. **(10 Marks)**

5 a. Apply Gauss-Jordan method to solve the equations $x + y + z = 9$, $2x - 3y + 4z = 13$, $3x + 4y + 5z = 40$. **(10 Marks)**

b. Use triangularisation method to solve the system of equations $3x + 2y + 7z = 4$, $2x + 3y + z = 5$ and $3x + 4y + z = 7$. **(10 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

6 a. Find the inverse of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix}$ using partition method and solve the equations $x + y + z = 1$, $4x + 3y - z = 6$ and $3x + 5y + 3z = 4$. **(10 Marks)**

b. Apply House-Holder's method to reduce the matrix $A = \begin{bmatrix} 1 & 4 & 3 \\ 4 & 1 & 2 \\ 3 & 2 & 1 \end{bmatrix}$ to the tridiagonal form. **(10 Marks)**

7 a. Give the properties of linear transformation. If $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ in $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, Suppose T is a linear transformation from R^2 to R^3 for $T(e_1) = \begin{bmatrix} 5 \\ -7 \\ 2 \end{bmatrix}$ and $T(e_2) = \begin{bmatrix} -3 \\ 8 \\ 0 \end{bmatrix}$, find the image of an arbitrary x in R^2. **(10 Marks)**

b. For $T(X_1, X_2) = (3X_1 + X_2, 5X_1 + 7X_2, X_1 + 3X_2)$, show that T is one to one linear transformation. Does T map R^2 onto R^3. **(10 Marks)**

8 a. Find the equation $y = \beta_0 + \beta_1 x$ of the least squares line that best fits the data points (2, 1), (5, 2), (7, 3) and (8, 3). **(10 Marks)**

b. Discuss :
i) Gram-Schmidit process.
ii) Least square lines.
iii) The general linear model.
iv) The orthogonal basis $\{V_1 \;\; V_2\}$ for ω, when $\omega = \text{span}\{x_1 \;\; x_2\}$ with $x_1 = \begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix}$, $x_2 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$. **(10 Marks)**

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